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Mathematical Model for Defining the Optimum Choice of a Means of Transportation

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Abstract

This model describes a mathematical way for finding the optimum choice between the means of transportation, before any travel. The optimum choice includes not only the cost, but also many other elements of a travel. Basically the model refers to trains, planes, cars, buses and ships, but it can also be used in the same sense for many other fields, in order services to be compared.

Keywords: Factors, Optimum, Travel, Cost, Model, Transportation.

1. Introduction

This model is based on the thought that not only the ticket price but also some other factors affect the means of transportation that people choose, before a travel.

These factors may be physical or intangible. The physical ones include for instance the ticket price, the duration of the trip and the travel distance, whilst the intangible factors refer to the passengers' beliefs and the time lag. Both type of factors modulate a travel and produce useful conclusions for the passengers, under specific mathematical working. Meanings such as time lag, fatigue, safety, comfort, are being transformed into real principles (numbers) and as a result they can reform the already existent travel cost.

The interesting point here is that each passenger is influenced by intangible factors but no one pays attention to this.

2. Main Text

2.1 Physical Factors

Physical factors (phf) are the basic principles of the travel.

i) The duration of the travel (dtphf)

ii) The ticket price or price and cost (pcphf)

iii) The means of transportation (mtphf)

iv) The number of travellers, travelling together (ntphf)

v) The running speed of the means of transportation (rsphf)

vi) The travel distance (tdphf)

There is an analysis of physical factors at the next stages of this research.

2.2 Intangible Factors

These are some factors that influence the travel but they must first be modified into real principles, in order to be compared and connected with the physical factors and therefore results to be produced. The intangible factors are:

i) The comfort (cf)

ii) The discomfort and time lag (dtlf)

iii) The travel safety (tsf)

iv) The service and personal beliefs (spbf)

The intangible factors are influenced by the reason of the travel, which is an indirect intangible factor. The introduction of the intangible factors to the travel is the innovative thought of this project. The inclusion of these factors to the travel via the ticket price produces a new ‘ticket price’ i.e. a new cost of the travel, as explained in the next chapters.

2.3 Common Scale

A common scale has been used from 1 to 100, in order all the factors to be compared and connected with the travel. This scale is ideal and necessary because:

i) A scale is needed for the reduction of all factors to it.

ii) The range of the recommended 1 to 100 scale is neither small nor big.

iii) The comfort and sound insulation table (table 1) is separated into 10 situations with ranking from 1 to 100 and this separation is complete. It can cover every travel situation.

iv) The mathematical equations at the comfort paragraph (paragraph 2.5) give results between 1 and 100 by themselves.

v) This scale does not influence the final results because in the end every result is being divided by 100.

vi) The 100% meaning is familiar to most people.

2.4 Physical Factors Analysis

The physical factors (phf), affect indirectly the travel .The dtphf affects dtlf and spbf. The pcphf affects all the intangible factors while the ntphf affects mostly the spbf. Finally, the tdphf affects all the intangible factors.

Physical factors are unique for every travel and it is impossible for them to be scaled in a scale because they are real. After all, it is not necessary.

An intangible factor analysis one by one follows.

2.5 Comfort Factor Analysis

The comfort factor (cf) is an intangible factor and as a result it must be modified into a number in order to be scaled in the 1 to 100 common scale.

At this paper, comfort is made of two acknowledgements.

i) Comfort is the free moving space for a passenger in a vehicle or vessel i.e. it is the free square meters of the vehicle (figure 1).

ii) Comfort is the sound insulation of a vehicle and the condition of the seat during the travel i.e. it's ergonomically nature and behaviour.

The first acknowledgement is called result of area factor (raf) while the second one represents the seat and sound insulation factor (ssif). The raf is unique for each vehicle and results by dividing the greater area between the compared means of transportation to each area of the compared means of transportation according to the following mathematical formula. The ssif results from table 1.

$$raf_i = \frac{area_{\max}}{area_i} \quad \textbf{(1)}$$

The half rate of raf and ssif gives the comfort factor as a number according to the following mathematical formula.

$$cf_i = \frac{raf_i + ssif_i}{2} \quad \textbf{(2)}$$

2.6 Discomfort and Time Lag Factor Analysis

In many travels, it is difficult for someone to study, work or make his time productive. Except for the time lag, the passenger is also being harassed because of the duration of the travel. The more one travels the more is being harassed.

The expression of the time lag and the fatigue into a number is the discomfort and time lag factor (dtlf) and as an intangible factor, it must be modified in the 1 to 100 common scale. A thought for achieving this follows.

At the first hour of a travel let it be that dtlf is 1 unit. The second hour of the travel the dtlf2 is the dtlf of the first hour plus dtlf of the second hour. At the third hour of the travel the dtlf3 is dtlf1 (first hour), plus dtlf2 (second hour), plus dtlf3 (third hour) and that makes 4 units. Judging by this, it results a geometrical progress expressed by the dtlf – time duration diagram (figure 2).

The diagram expresses the mathematical formula (3) which is the fatigue at a specific time point.

$$dtlf = 2^{T-1}, (T \geq 0) \quad \textbf{(3)}$$

At the same diagram results the mathematical formula (4) which is the accumulated fatigue of all hours.

$$\int_0^T 2^{T-1} dt \quad \text{or} \quad \frac{1}{\ln 2}\left(2^{T-1} - \frac{1}{2}\right) \tag{4}$$

The dtlf diagram and the mathematical formulas produce useful and interesting information.

i) The dtlf at time (0) is not (0) but 0.5 according to the mathematical formula (3), i.e.

$$dtlf(0) = \frac{1}{2} \tag{5}$$

Hence, there is a pre-fatigue situation for every passenger. This can be explained with the next thought. The situation of a person changes because of the travel. He/she will be a traveller in the near future. There is a phase, maybe few hours or even days before a travel that affects every traveller. At this phase (let it be pre-travel phase) the person thinks of the information of the travel that is safety, travel conditions, time of arrival, if anything is missing, ticket availability, on line ticket booking, baggage preparation etc. According to this, the pre-fatigue is caused because the time flow before a travel is interrupted by the travel. There is so a not continued time; a time situation changeability. The 0.5 units are the result of those conclusions in the dtlf – time duration diagram.

ii) It is not necessary that all the means of transportation have the same dtlf – time duration diagram. In cars for example, if there are repeated stops in a trip, the diagram will be repeated curves and that decreases the dtlf result, because the final dtlf is the integration of those curves. Figure 3 presents the dtlf diagram form, in a hypothetical travel by car with one-hour stop, every two hours of travelling. This fact dissociates cars from the other means of transportation. It gives them an extra advantage. Also it reveals the time that a traveller needs in order to relax from the travel, in other words, in order the fatigue reaches 0.5 units, according to the diagrams.

iii) It is proved that a travel causes two forms of fatigue. There is a difference between the fatigue at a specific time and the accumulated fatigue. This difference can be estimated according to the mathematical formulas (3), (4). That fact not only proves the validity of dtlf but it also generates two questions. Does fatigue have a relation to travel illnesses such as dizziness, jet lag or nausea? Which of the two forms plays a greater role in that? Is the dtlf factor the numerical expression of the travel fatigue that for so many years many scientists have been trying to estimate?

2.7 Travel Safety Factor Analysis

The travel safety factor is not only an important intangible factor but also something that all travellers do care about. Of course, it is impossible for a means of transportation to be absolutely safe.

This reality generates the travel safety factor (tsf). Through valid statistics, the total number of passengers can be compared with the total number of deaths because of accidents, at the same period.

For example, according to the National Hellenic Statistic Department rail passengers in the route Athens – Thessaloniki, from the year 1990 to 2002 were 3 millions. At the same period 12 passengers were killed in this route, at railway accidents (train derailments, fires on trains etc). That gives 250,000 rail passengers per 1 death. The same reasoning could operate for planes, ships, cars etc. That conclusion is described by the following mathematical formula which includes total number of passengers (tnp) and total number of deaths (tnd) at the same period (ΔT).

$$tsf_i = \frac{tnp_i(\Delta T)}{tnd_i(\Delta T)}, (tnd_i \geq 1) \quad \textbf{(6)}$$

The mathematical formula (7) gives the compared results between the means of transportation, in the 1 to 100 common scale. This is called result of travel safety factor (rtsf).

$$rtsf_i = \frac{tsf_{\min}}{tsf_i} \cdot 100 \quad \textbf{(7)}$$

2.8 Service and Personal Beliefs Factor Analysis

This is the last intangible factor and the most subjective. The half rate of the following situations and service beliefs is the number that expresses spbf.

Every passenger gives marks to service (s), nausea (n), claustrophobia (cph), height phobia (hph), dizziness (d), other phobia (oph), environment pollution care (epc), of a means of transportation. The marking scale is the 1 to 100 common scale. Then the following mathematical formula gives the result of spbf.

$$spbf_i = \frac{s_i + n_i + cph_i + hph_i + d_i + oph_i + epc_i}{7} \quad \textbf{(8)}$$

The seven things for ranking include the basic phobias, the environmental sensitivity of the traveller and the service level, which includes for example the gentle personnel, the ticket availability etc. All the above can be found in the spbf diagram (figure 4) and depend on the chosen means of transportation.

2.9 Figures and Tables

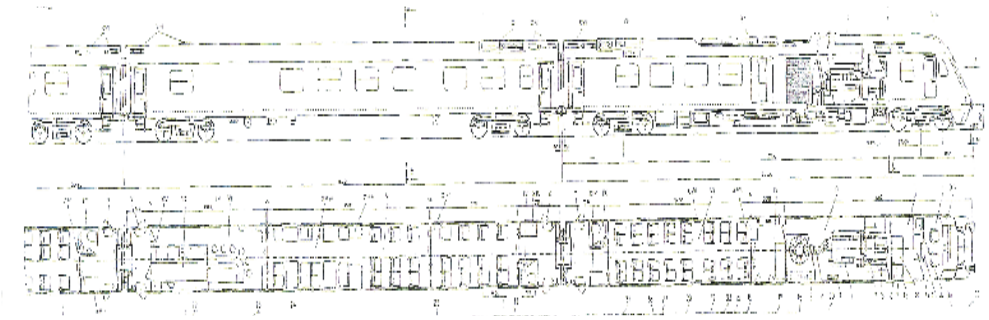

Figure 1. *Inside area of a train vehicle. The free square meters can be estimated by multiplying the length with the width of the whole train.*

Table 1. *ssif – 10 travel situations*

1	2	3
Sound insulation-seat condition	**factor**	**General characterization**
Absolute insulation and seat condition	1-9	First rate
Excellent insulation and seat condition	10-19	Excellent
Good seat condition and sound proof	20-29	Very well
Good travel conditions	30-39	Good
Medium travel conditions	40-49	Medium
Noisy travel and normal seat condition	50-59	Affordable
Noisy travel and bad seat condition	60-69	Bad
Noisy travel and dirty seat	70-79	Very bad
Very noisy travel and very dirty seat	80-89	Insufferable
Overcrowded, noisy travel and no seat existence	90-100	Sleazy

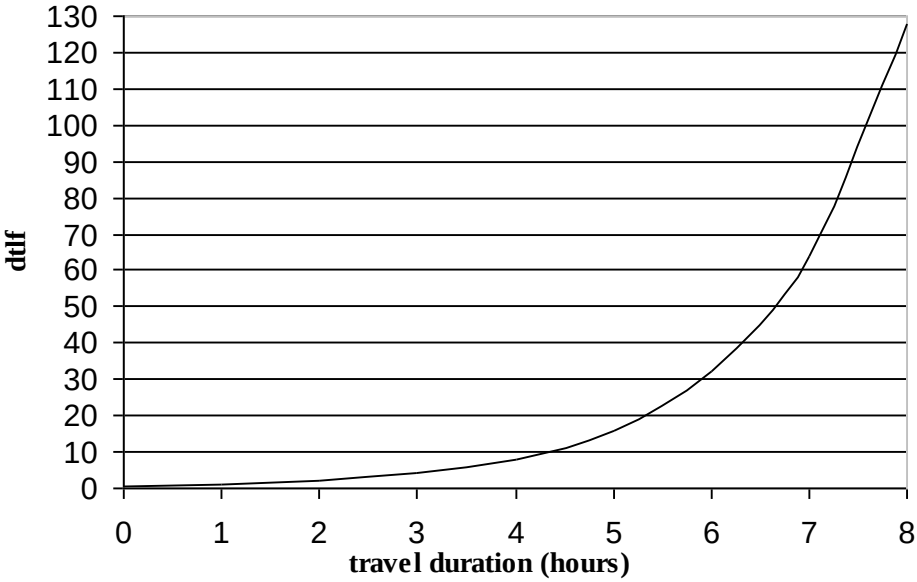

Figure 2. *Diagram of dtlf-travel duration*

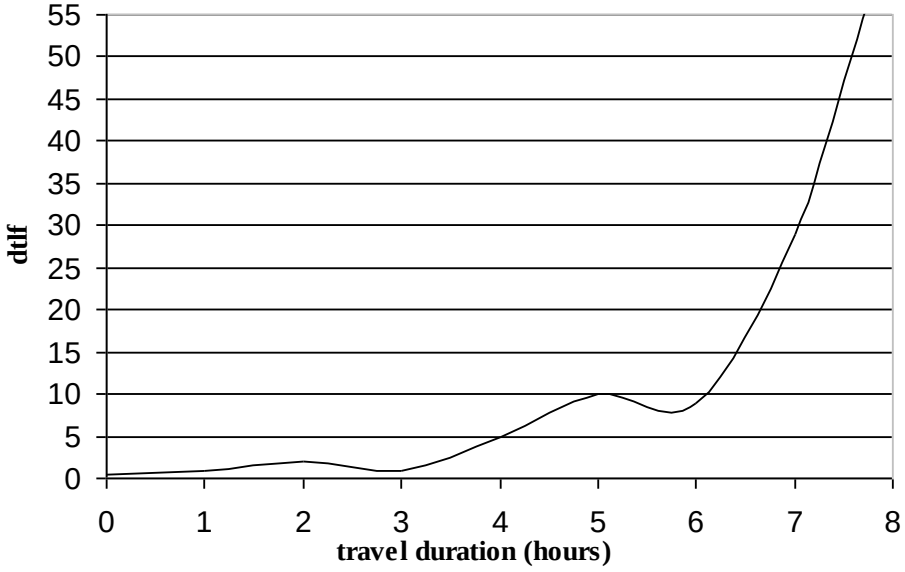

Figure 3. *Diagram of dtlf-travel duration in a car travel with stops*

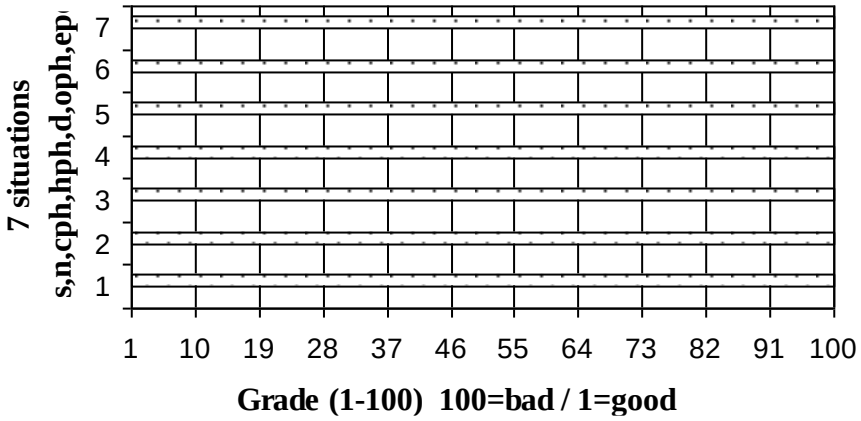

Figure 4. *Diagram of spbf, with seven situations for ranking*

2.10 Conclusions

There are two different ways of thinking in order the final result to be estimated.

The first way (method i), covers those who want to follow the logic of this work while the second (method ii) covers those who have strict personal beliefs for all the factors. Both methods produce logical and objective results.

i) After having the results through the mathematical formulas and diagrams, a total final factor cost ($tffc_i$) is being produced by the following mathematical formula.

$$tffc_i = pr \cdot \left(1 + \frac{cf_i + dtlf_i + rtsf_i + spbf_i}{100}\right) \quad \textbf{(9)}$$

The ticket price (pr) is being multiplied with the intangible factors and the result is the new modified price which can be compared with the modified prices of the other means of transportation. The lower the price, the better the transportation choice. Perhaps, in some cases, the modified costs are approximately the same between the compared means of transportation. When this happens, the most suitable choice is the means of transportation which has the smaller difference between its ticket price and its modified price. The comparison of course, must be done for the same route, which means for the same destination.

ii) The mathematical formula (10) includes bj factors, which are the gravity numbers.

$$tffc_{ii} = pr \cdot \left[1 + \left(4 \cdot \frac{b_1 \cdot cf_i + b_2 \cdot dtlf_i + b_3 \cdot rtsf_i + b_4 \cdot spbf_i}{(b_1 + b_2 + b_3 + b_4) \cdot 100} \right) \right], (b_j > 0) \quad \textbf{(10)}$$

Bj factors make an intangible factor more important than the others. When all bj factors are zero, the mathematical formula (10) is useless and this means that the travel cost is what a traveller pays for his ticket. In other words the factors of this research are not important or do not play any role at the modification of the travel. When all bj factors are 1, this means that each intangible factor has the same importance with the others. In any other case the traveller chooses his personal gravity numbers and the total final factor cost ($tffc_{ii}$) is the result of the mathematical formula (10). For better results, the range of bj numbers must be small, for example the maximum bj could be the number 10. Also, the comparison must be done for the same destination.

The final cost, either $tffc_i$ or $tffc_{ii}$, includes the intangible factors and it is the real travel cost, as this work protests.

3. References

AEGEAN AIRLINES. (2006). 'Tickets and destinations'. Leaflet for passengers with information, pp. 1-2.

Chrisoulakis I., Dimitriou D. (2004). Chapter 1. Transportation management and European networks: 1.1 transportation designing. Academic book for Transportation Systems of TEI Athens, vol. 1, pp. 1-18.

Hellenic Statistic Department. (2004). TRANSPORT AND COMMUNICATION (1990-2002). Copies for transports of the Hellenic Railways Organization 1990-2002. Copies for deaths at railway accidents 1990-2002. Number of pages: 4.

Iliadi K., Skouta I. (2005). Marine Department of the BLUE STAR FERRIES Company. Fax with information about the size and the operational cost of the ship BLUE STAR ITHAKI, number of pages: 1.

S.F.S. (Silogos Filon Sidirodromou). (1994). RAILTRACK. 'The new INTERCITY trains are coming in Greece'. Magazine of the Railway Friends Club, vol. 10 December 1994, pp. 40-41.

Tsimberdonis A. (2004). Operational Research. Academic leaflet for the lesson of Operational Research at the school of Technological Applications, Department of Civil Engineering Technology, TEI Athens. pp. 2-12.